Multistability –
More than just a Freak Phenomenon

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Abstract: A classification of forms of multistability reveals that multistability is not confined to experimental settings but a pervasive, albeit inconspicuous phenomenon also in complex scenes and events.

Analyzing the conditions of multistability and its relation to spatial perception results in the notion of parallel, cooperative as well as competitive processes contributing to the emergence of a stable spatial percept.

The epistemological implications of such an approach are discussed and termed 'Interactive Realism;' according to which the processes underlying the 'constructive mind' have evolved under the constraints of the transphenomenal (physical) world.

1 What is multistability about: The real world or the laboratory?

If one looks at the astounding examples of multistability, e.g. Jastrow's duck/rabbit (Fig. 1) which has influenced Wittgenstein in his Philosophical Investigations or the looking glass/skull (Fig. 2), or if one experiences the involuntary switches in the perception of the Necker cube and compares these with the apparent stability of everyday perception one tends to write off these phenomena as freak phenomena albeit interesting, perhaps telling as much about the processes of perception as the Manierist anamorphic pictures (see e.g. Leeman, Elffers & Schnyt, 1976) tell about Renaissance art.

This point has been made in the framework of Ecological perception according to which either illusions happen only if the information in the provided stimulus is underdetermined (Michaels & Carello, 1981, p. 180, 181) or 'the perception is not in error' but the researcher who erroneously equates measures of physical energy with reality (Michaels & Carello, 1981, p. 71).

Regarding their venerable status in the history of perception calling phenomena like Rubin's vase, Jastrow's duck/rabbit or Necker's cube freak phenomena might ring of facetiousness but if one takes into account the fact

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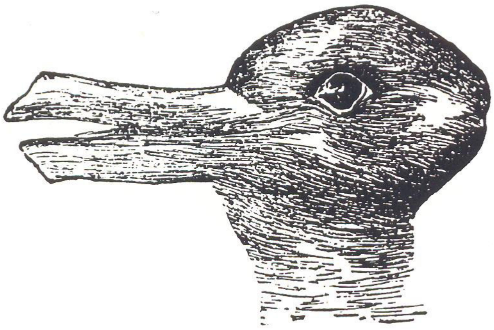


Fig. 1 Jastrow's duck/rabbit form

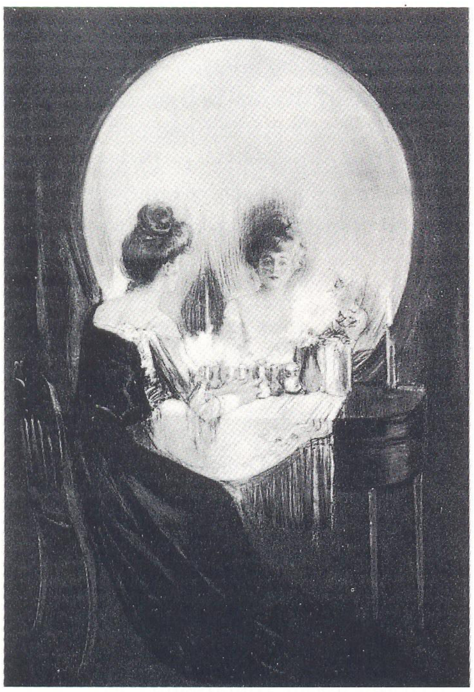


Fig. 2 The looking-glass/skull picture

that these phenomena exhibit multistability only under very specific conditions or in the case of the wire cube or wire staircase seem to lack ecological validity, one is tempted to doubt their importance for more general rules of perception. However, the field of multistability is diverse and therefore before starting a detailed analysis, a classification of types of multistability might be helpful.

The distinctions I have made are not as clear cut as it appears, for instance, the major characteristic of the geometrical category, namely the induction of 3-dimensionality in 2-D displays, can also be found, albeit to a lesser degree, in the other categories: In the looking glass/skull example as well as in the 'View of a City' the semantic and/or the Fig.-ground disambiguation induce depth. However, as Brandimonte & Gerbino (1993) have demonstrated, semantic priming can be used to classify types of multistability because only for semantically ambiguous forms does priming cause the perception to be uniquely specified. That a combination of different types of multistability can achieve striking effects, demonstrate Bradley & Petry (1977) in their Necker cube viewed through peep-holes in a white surface before a black background or in front of 8 black circles on top a white background; if the second view is taken, virtual contours appear as in Kanizsa (1955).

Table 1 Classification of types of multistability

Category	**Example (only those examples which are depicted in this article)**
semantic	rabbit/duck (Fig. 1), looking glass/skull (Fig. 2)
Fig.-ground	"View of a City" after Metzger (1975) (Fig. 3)
illusory contours	Bradley and Petry's view of a Necker cube (Fig. 4)
geometrical multistability	
(a) depth	the Necker cube (Fig. 4) Japanese patterns (Fig. 5)
(b) concave-convex	the cupola of S. Giovanni degli Eremiti (Fig. 6)
	Roman-Byzantine tilings (Fig. 7)
(c) classification	non-periodic Penrose tilings (Fig. 8)

My goal is to show that by the observable regularities in the domain of geometrical multistability we gain insights into the tuning of perception for picking up objects and spatial relationships between these objects and the perceiver. The multistability of the Necker cube will serve to define conditions for spatial impressions from flat graphical designs.

That multistability is more than a negligible fringe effect in visual perception becomes obvious if one analyzes the amount to which the information giving rise to a stable percept really specifies the seen objects and their spatial arrangement in regard to each other and to the viewer. Figure 9 gives

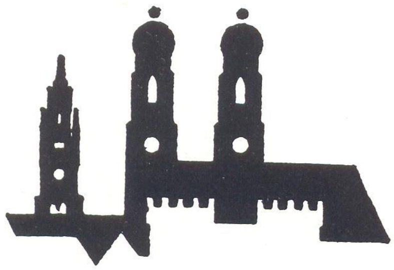


Fig. 3 "The View of a City" after Metzger, 1975

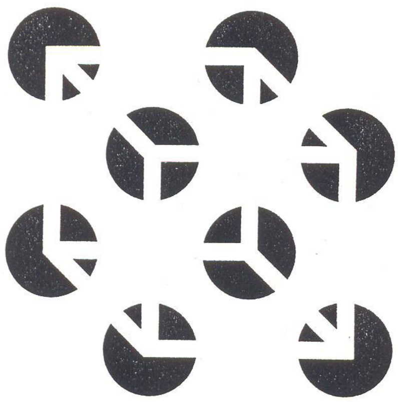


Fig. 4 A Necker cube seen through eight peepholes or before eight black circles (after Bradley & Petry, 1977)

the view of a cathedral together with 3 different groundplans (out of many other possible once) compatible with this view.

However, this 'real-world' multistability in a complex situation remains unnoticed because only the interior with a rectangular groundplan is 'seen'. This can be interpreted as the result of two complementary processes in perception, that of stability and its tendency towards simple forms (Wertheimer, 1912) and that of singularity which specifies uniquely the position of the viewer relative to the perceived objects. Kanizsa and Luccio (1986) were the first to point out that the term 'Prägnanz' in Gestalt theory really is ambiguous; Zimmer (1991) in commenting them has shown that the relation of singularity and stability is that of a complementarity: "The second aspect of the complementarity, namely that of interaction between local vs. global optimization, can best be exemplified in the field of spatial perception where the forked effect of local optimization (stability leading to the transformation of ellipses into circles. of arbitrary rectangles into squares etc.) and the uniqueness (singularity) of the point of view give rise to a stable

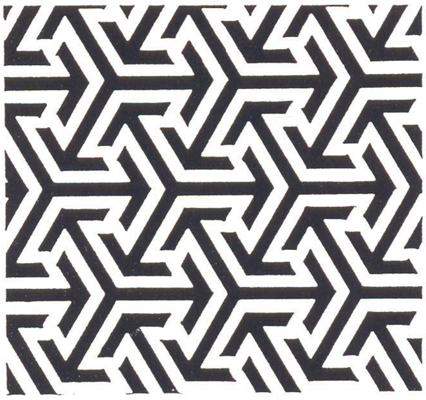


Fig. 5 Japanese patterns inducing a depth effect

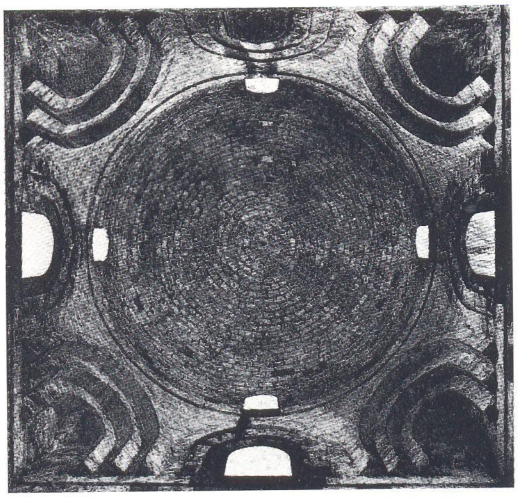


Fig. 6 The cupola of S. Giovanni degli Eremiti in Palermo exhibiting a concave-convex switch in the upper pendentives of the main cupola

image of the surrounding world despite the fact that any given projection can originate from a multitude of spatial arrangements. LaGournerie in the last century (see Fig. 9) and Ames since about 1935 have shown this and how local orthogonalizations or symmetrizations together with the singularity of the viewing point result in unique spatial impressions even if they contradict 'known facts' as e.g. the relative height of people in the Ames-room. One prediction from this assumption that it is the tension between a global tendency towards stability and the sensitivity to local disturbances which generates the impression of space, is that this impression should be strongest if the forked

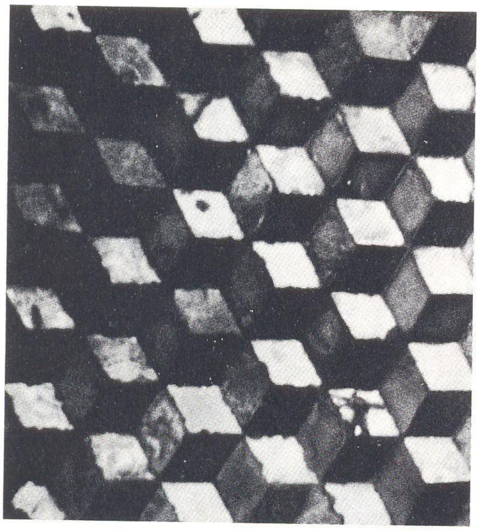


Fig. 7 A Roman-Byzantine mosaic (about 500 AD)

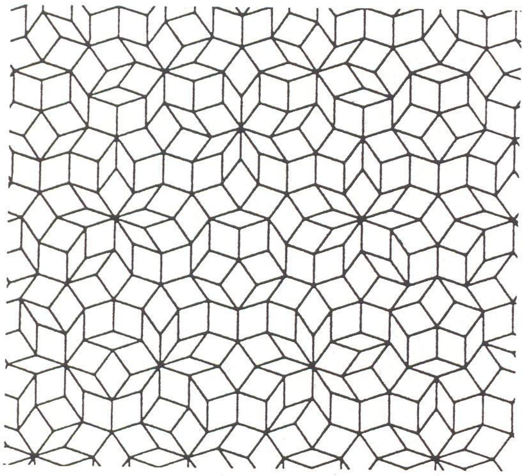


Fig. 8 A non-periodic Penrose tiling

effects are about equal. Since the effects cannot be measured directly, this hypothesis can be tested indirectly by showing that in 2-dimensional drawings the strength of the spatial impression is not maximal for drawings that obey perfectly the rules of perspective but for those which form a compromise between stability of partial forms and perspective distortions" (Zimmer, 1991, pp. 277 – 278).

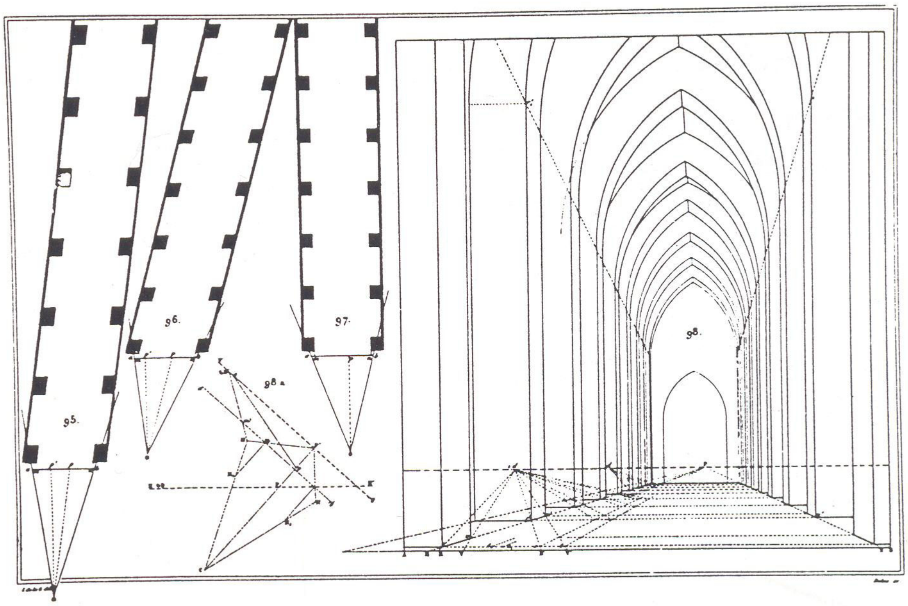


Fig. 9 A view of a cathedral with 3 compatible groundplans (after LaGournerie, 1859)

That the relationship between stability and singularity is a complementarity can be seen from the fact that in the perception of simple stimuli, for instance angles or triangles, even minute deviations from orthogonality or triangularity are detected immediately, which Goldmeier (1982) attributes to the effect of singularity, but that in remembering the same stimuli (e.g. triangles) there is a strong tendency for nearly all stimuli to be identified with the equilateral or orthogonal ones (Zimmer, 1991), an effect which can be modelled according to the laws of gravitation, that is, a process with a tendency towards most stable configuration. This implies that with any potential landscape characterized by sinks, which specify the control parameters in the synergetic approach, there is associated an inverse landscape where these points of stability become singularities (Fig 10 a, b).

One implication of such a model is that the traditional separation of perception and memory should be abandoned; instead of it a unitary if complex coping of organisms with their environment is postulated.

The duality of perceiving objects as they are and at the same time perceiving the spatial relations between the perceiver and the perceived seems to be fundamental for the process of perception. Nevertheless, in experimental research this duality is usually dissolved: Either object and/or feature recognition or distance, time-to-contact or spatial perception are analyzed. My hunch is that this dissolution is partially due to the fact that it is difficult to construct stimuli which are complex and exactly analyzable and

a b

Fig. 10 Two complementary potential landscapes: (a) after Haken (1991); (b) its inverse

controllable at the same time. This is one of the reasons why I think that inherently complex stimuli such as multistable forms and perspective drawings offer an opportunity for an experimental analysis of space perception without dissolving this fundamental duality.

The epistemological approach most in line with this line of argumentation might be termed Interactive Realism, because it accounts well for the apparent order in the perceived world.

2 Perception and epistemology: Do we pick up what is already there or do we construct it out of our heads or both?

Before delving into the questions concerning the order in nature of in our perception of it, some words about Realism seem to be necessary. In the plainest sense, Realism is the epistemological position which claims that the objects we see, feel, remember and talk about have a correspondence 'out there' in the physical realm, that is, the mind is intentional. In this sense the entire pre-Socratic philosophy plus Aristotle and even Plato adhered to a realist position. The extreme counterposition of solipsism is so contrary to the world of everyday experiences that even Idealists tended to subscribe to a primitive Realism as a kind of epistemological truism.

What advances the Realist position toward more specific epistemological theories are the questions about natural categories (e.g. humans), about natural laws (e.g. can we say something about gravity that refers to something as real as an individual stone?), about the relation between mental representations of objects and the physical objects and about persistence over time. If one answers all these questions in the affirmative, one arrives at the position of Direct (or derogatorily called Naive) Realism as championed by J.J. Gibson:

"It seems to me that these hypotheses (namely (1) the existence of stimulus information, (2) the fact of invariance over time, (3) the process of extracting invariants over time, and (4) the continuity of perception with memory and thought make reasonable the common sense position that has been called by philosophers direct or naive realism" (Gibson, 1967, p. 168).

Gibson discards alternative Realist positions, for instance Holt's, as circular in their support for this common sense position because they regard perception as sensation-based; in contrast, his main 'new reason for Realism' is the notion of the information-based perceptual system. Appealing as this common sense position is, one should nevertheless ask oneself why this position has come to be questioned at all. I want to do this in a paradigmatic fashion in regard to the Realism-Nominalism debate in Scholastic philosophy because it can be shown that this old debate is far from outdated.

The core question of this debate is about universals: Are universals real, independent of the mind, as William of Champaux (1070 – 1121) claimed, or are they nomina, nothing but an exhalation of the voice (flatus vocis), as Roscelin (1050 – 1125) objected? It is apparent that William of Champaux' claim cannot be generally true, there exist without doubt classes of objects made up by a specific mind or being the result of a cultural convention. But on the other hand to deny the reality of natural categories implies that no planned interaction with nature (including science) is possible because then only individuals at specific times and places are real and any predictions about classes of individuals or about individuals over time have no basis in reality.

This problem is stated most succintly by Lichtenberg, a German physicist and philosopher of Enlightenment:

"Everything that we as humans recognize as being real, is real for humans. Otherwise, if it were no longer possible to be able to judge every natural constraint as reality, it would be impossible to think even in terms of concrete principles."(Georg Christoph Lichtenberg from "Bemerkungen" in the years 1766 and 1799).

Modern versions of Roscelin's Nominalism underly Whorf's thesis of cultural relativism and, for instance, Quine's postulate that classes are generally abstract and not real entities, is genuinely nominalistic. Furthermore the 'language of thought' (Fodor, 1974) or the computation approach (Pylyshyn, 1984) to human information processing are in a way nominalistic as well as van Fraassen's or Putnam's antirealism. A side effect of the debate shows in modern logic where the universal quantifier (as in "all men are mortal") is paraphrased (as in "for any x, if x is a man, x is mortal").

Already Abelard (1079 – 1144), William's pupil, and later especially Roger Bacon (1220 – 1292) and Ockham (1285 – 1349) have concentrated on questions like 'what constitutes natural categories' and 'what is the relation between the concept of a class (the representation in the mind) and its counterpart in the physical world?' Abelard approached the problem from an ontological Platonic angle: There is no common essence in a class but a common idea present when God created it, that is, from God's point of view, universals are 'ante rem' but from the human perspective, they are 'post rem'. In contrast, Roger Bacon and Ockham searched for empirical criteria and arrived in the end at a position of conceptual empirism: Bacon's "Deficiente sensu deficit scientia" postulates that the concepts of science (as pure cases of natural categories) have to be based on observables. Ockham gave this position an even more radical twist when he denounced the theories of mediated perception (especially that of Duns Scotus) and claimed that perception is direct, making him a predecessor of Gibson and modern Direct Realism.

A further attempt to save Realism from the Nominalist attack is concerned about the relation between the representations in the mind and their objects in the physical world leading to the Representational Realism of J. Locke; the shortcomings of which were resolved in Santayana's 'Critical Realism' (1920) distinguishing the phenomenological world and its transphenomenal counterpart.

An early attempt to define non-mentalistic processes inducing a self-organisation of the phenomenal world under the same constraints as present in the physical world was W. Köhler's field theory (1920) which set the frame for the Gestaltist theory of perception.

For terminological clarity, I want to point out that I use the term 'physical world' (not 'world of physics') equivalent with the term 'transphenomenal world' as introduced by Santayana (1920) in his treatise on 'Critical Realism'. In this position the Nominalist fallacy is refuted according to which the physical world (res) and the cognition operations of the human mind upon it (Roscelin's 'flatus') can be neatly separated. The tradition of that line of argumentation can be found in Hume's theory of perception, refined by Kant in his theory of schemata which link the 'images' of the physical world and the categories of thinking. However, Kants interactionist position on perception has not been very influential for the development of psychological theories of perception because starting with Helmholtz' hierarchy of perceptual processes: Sensations, unconscious inferences, conscious cognitions, the Nominalist tradition of separating sensations as directly influenced by the physical world and perceptions as the cognitively processed sense informations is very alive albeit disguised in modern theories of perception, for instance, in Marr's (1982) or Pylyshyn's computational approach (1984).

The problems about the relation between our mental representations and the world 'out there' have not lost their importance and some of the riddles of Realism are far from solved (e.g. how is the information produced that constitutes Gibson's ecological perception which allegedly solves these problems?) but also a radical Nominalism is untenable, or at least implausible and impractical for science. Therefore further investigations into Realism are necessary. Multistability can be regarded as a paradigmatic approach to the question how the order in the phenomenal world is related to that of the physical world and by doing this, one arrives at a position which might be termed Interactive Realism; a position claiming that for the perceiver the order of the world which is in the mind is a constituting element of this world itself. Contrary, however, to the position of Radical Constructivism (Maturana, 1982) it is claimed that the meaning producing mechanisms of the mind as – for instance – perception have evolved under the constraints of the physical world and in interaction with it.

How the external world might impose constraints on perceptual mechanisms has been modelled by Rechenberg's evolutionary algorithms (1990) and by means of cellular automata (e.g. Li, Packard & Langton 1990 analyze the behavior of cellular automata dependent on the parameter λ.); one consequence of this kind of modelling is that the resulting solutions are very often thus that a single, albeit complex, algorithm cannot be found which describes the entire function of the mechanism correctly but that two or more competetive, complementary, or cooperative algorithms in parallel are necessary. In some cases where the coupling of the external world and the behavior of an organism might even be described by a single algorithm, as for instance the τ-algorithm for the time-to-contact control of action, it turns out that the observable behavior relies on more than one but simpler algorithms; Borst and Egelhaaf (1992) have shown this to be the case in the landing behavior of house flies.

Apparently such a coupling of external constraints and perceptual processes is more stable against perturbations and at the same time more sensitive to singularities, than entirely internally or externally driven can be. The prototypical example of the evolution of perceptual mechanisms under the constraints of the external world (physical reality) is the sensitivity of the eye (in humans as well as in most animals) to those electromagnetic wavelength with especially high energy in the sunlight which reaches the surface of the earth (see Land & Fernald, 1992, for an overview). Any sense organ tuned to other wavelengths would need a much higher sensitivity to result in a comparable discriminative power (acuity). Interestingly, the distribution of sensitivity for colours mirrors not the distribution of wavelengths in sunlight but the distribution in reflectance of objects in the environment. Shepard (1990) has sketched an evolutionary process for trichromatic vision pointing out that it achieves a maximum for the robustness-informativeness trade-off (Zimmer, 1982) in perception. It seems plausible to assume that this trade-off plays a central role in coordinating the physical world (reality) and the mental representations of it.

What I have termed Interactive Realism can already be traced back to Aristotle's theory of perception. In his Metaphysics, he stresses the point that 'seeing' means 'seeing something', that is, it is an intentional act not a mere registering of what is phenomenologically external. However, in the same context he mentions the importance of the viewer: "[seeing is always] the seeing of the person whose seeing it is."

Following this line of argumentation, one would agree with the Radical Constructivist (Maturana, 1982) position according to which percepts are not projections from the physical world but constructions of the perceiving organism reflecting its intentional position as well as its mechanisms of perception. However, and here I think the Realist stance cannot be parted with, the perceptual mechanisms also have evolved under the constraints of the physical world. In such a way already Koffka (1935) has answered to the question "Why do things look as they do?" (p. 76) namely because in perception "...the processes organize themselves under the prevailing dynamic and constraining conditions" (p. 105) and only such processes survive which optimize the interaction of the organism with its environment.

The position of Interactive Realism in regard to space perception has been circumscribed by Koenderink (1990) as follows: "... homunculi or God-given local signs are not required in a formal scientific understanding of the geometrical expertise of brains, that is, the apparent ability to generate efficacious potential action based on optical exposure. The brain can organize *itself* through information obtained via interactions with the physical world into an embodiment of geometry, it becomes a veritable *geometry engine*. Whether space is in the head, or the head is in space, remains undecided here. In the final analysis the distinction is scientifically meaningless anyway in view of the inherent circular nature of vital processes including optically guided behavior." (p. 8). Investigations into the multistability of wire cubes or related forms and into the bag of tricks used in visual arts to induce depth in flat displays will help to clarify the relations between the world of things (reality) and that of actions including perception (actuality).

3 The wire cube as a paradigm for space perception

In Fig. 11 six skeleton cubes are presented which differ in regard to their strength in inducing 3-dimensionality (apparent depth) and to the degree of multistability: In both regards, Fig. 11b induces the strongest effects because it has two equally strong semi-stable views which correspond to alternate spatial organisations.

Only slighly weaker is the effect in 11 a because here a third very transient state can be observed in which a 2-D pattern with a 45° axis of symmetry appears, this effect can be enhanced by rotating the axis of symmetry into a vertical position (see Zimmer, 1986). From an Empiricist point of view only 11d–f should give the impression of depth because only they correspond to real perspective projections with one, two, or three vanishing points; in contrast, however, 11a is a perspectively impossible view of a wire cube and

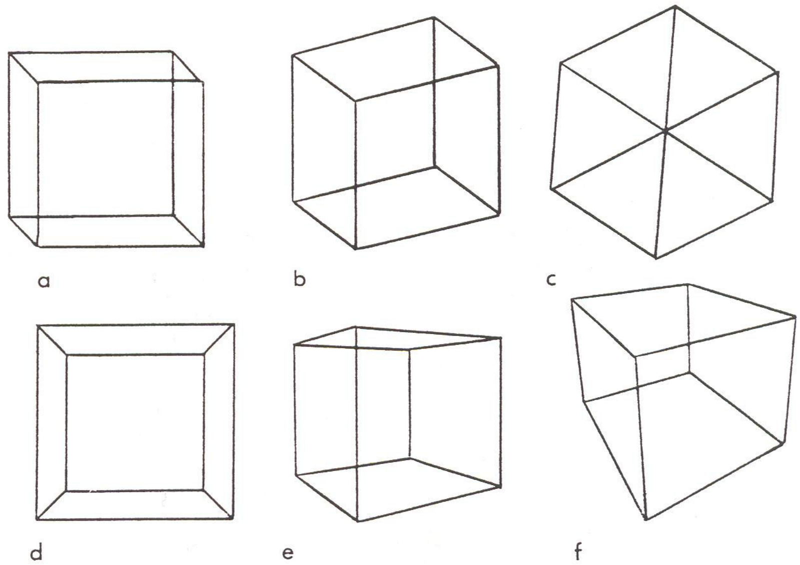


Fig. 11 Six wire cubes which all fulfill the general criteria for cubes (8 threefold vertices plus quadrupelwise parallel connecting lines) but differ according to the degree in which they obey the laws of projective geometry

11b as well as 11c are parallel projections without a vanishing point or one infinitely far off, that is, they are not possible in a finite environment. On the other hand, because of their unique perspective specification 11d–f should not induce multistability. As it turns out, they induce a much weaker effect regarding apparent depth than 11a and b – apparent from the fact that minor perturbations, namely, removing the vertices, destroy the 3-D effect - but they nevertheless give rise to bistability albeit with a bias towards the cube. This bistability seems to defy the Gestalt theoretic minimum principle (Köhler, 1920), according to which deformations of forms exhibiting 'Prägnanz' (singularity) are avoided by perceptual mechanisms (Koffka, 1935). However, practically none of the closed geometrical forms in 11e or f shows the high degree of symmetry typical for stable 2-D forms (Wulf, 1922); the cube is a 3D form with multiple axes of symmetry and therefore highly stable but none of these characteristic features is preserved in the perspective drawings: The lines are not parallel and of equal length, the angles are not orthogonal. At least partially, these features are preserved in 11a–d, where this preservation plus the breaking of symmetry induces the strong 3-D effect especially in 11 a and b. If, in contrast, the optimal symmetry of a 2-D projection of a cube exhibits maximal symmetry (6 axes of symmetry as in c and 8 in d), no or only a very weak and transient depth effect is induced.

But back to the startling effect of multistability in the unique projections of 11e and f: If one assumes that the forks and arrows at the vertices with their

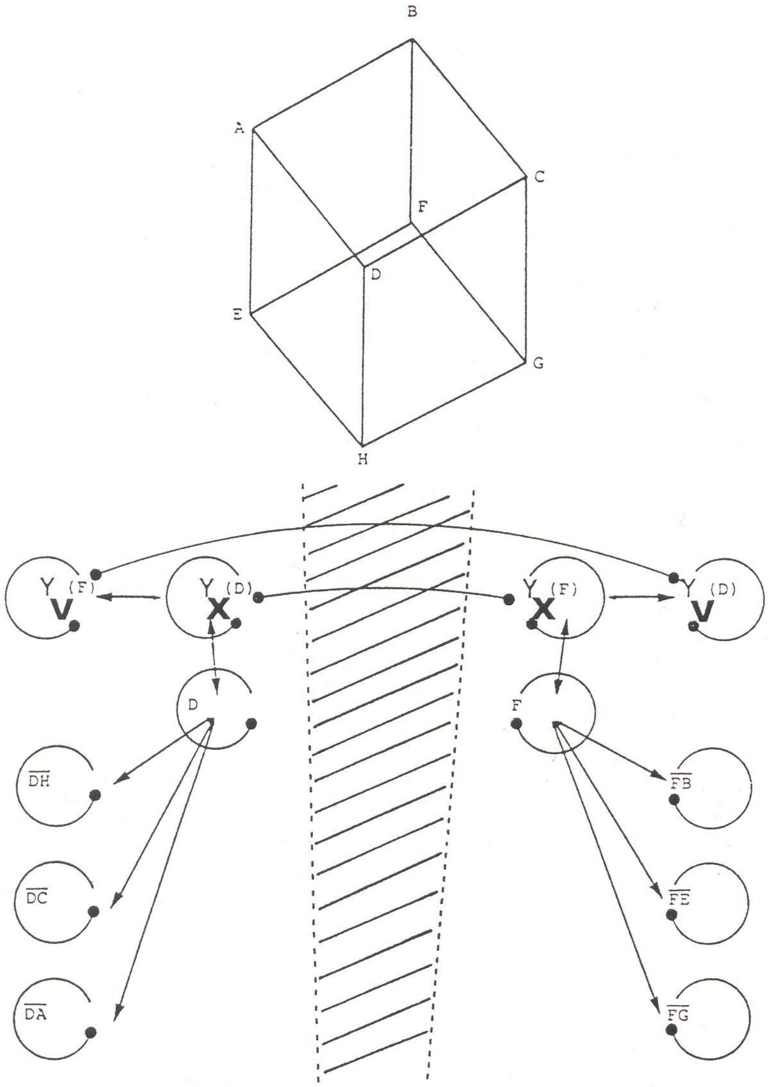


Fig. 12 Zimmer's (1989) network model for the distribution of attention during the inspection of a wire cube with excitatory and auto-inhibitory connections. Arrows indicate excitatory relations, dots inhibitory ones. Y_v and Y_x indicate fork junctions which are either concave (V) *or* convex (x). The edges indicated by DH and the vertices (capital letters) are always in front. Since only these vertices and edges are attended to, the others are regarded as inactive. The shaded area indicates the region where D and F cannot be discriminated and no depth effect and perspective reversal happen (Fig. 11c)

ambiguity of being either convex or concave can undergo selective satiation or, if modelled as a network, have auto-inhibition (Zimmer, 1989), the switching of only one node immediately triggers the switching of all other nodes into their alternate states (Fig. 12). What results, however, from this structural change is a form which fulfills Attveave's (1981) model of a generalized soap bubble, not as perfect as a cube but nevertheless convex.

Among the wire cubes of Fig. 11 there still remains one puzzling exemplar, namely, 11d which preserves many features of the cube *and* is a possible projection but nevertheless appears flat or as a picture frame but not as a cube. Perkins (1968) has used this instance to derive laws for the induction of 3-dimensionality in pictures:

(i) A fork juncture is perceived as the vertex of a cube if and only if the measure of each of the three angles is greater than 90 degrees.

(ii) An arrow juncture is perceived as the vertex of a cube if and only if the measure of each of the two angles is less than 90 degrees and the sum of their measures is greater than 90 degrees.

These laws predict correctly that Fig. 11d is seen as flat. However, if one breaks the symmetries as in Fig. 13, forms are generated which defy Perkins' laws, especially a and c.

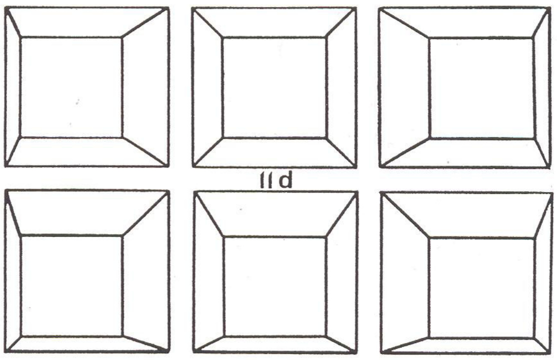


Fig. 13 Variations of the view of the wire cube in Fig. 11d with different degrees of symmetry breaking

The wire cubes in Fig. 13 cannot only induce a depth effect, they even exhibit multistability between a cube seen from the inside and a cut-off pyramid seen from outside. There is no clear cut preference for any of these perspective orientations because the subjects are about evenly split between an initial identification as a cube and a pyramid; afterwards subjects tend to stick to their initial percept. This is probably due to the fact that the biases for the convex form (pyramid) and for the most regular form (cube) are about equally strong. A similar 'real-world' example is the view of the cupola of S. Giovanni degli Eremiti in Palermo where the cupola is usually seen as concave but the pendentives tend to induce a convex orientation in spite of the knowledge about the constructive rules for building cupolas over square groundplans (see Fig. 6 above). A couple of lessons about multistability and its role in depth perception can be derived from this analysis of different projections of wire cubes:

(i) If the necessary conditions for perceiving a wire cube are given, namely 8 vertices with 3 connections each, plus convexity, apparent depth is induced if and only if the symmetry of the display is broken; an orientation of 30° relative to the fronto-parallel plane seems to be optimal.

(ii) The depth effect as well as the symmetry of the multistability are strongest if as many as possible features of the cube are preserved in its projection.

(iii) Experience in the sense that the subjects might have seen such a projection or in the sense that the rules of projective geometry are known and applied seem to play a negligible role.

These results agree very well with the results of a factor analysis of 'reversible-perspective drawings of spatial objects' by Hochberg and Brooks (1960). According to that study the apparent depth of a drawing depends on three factors: (i) simplicity vs. complexity (measured by the number of angles), (ii) good continuation vs. segmentation (measured by the number of line segments), and (iii) symmetry vs. asymmetry (measured by the relative number of different angles). What this factor-analytic approach implies, however, is the additivity of these components: "The greater the *complexity*, the *asymmetry*, and the *discontinuity* of the projection of a given tridimensional object in two-dimensions, the more three-dimensional it will appear. We may, in reality, be dealing with only one dimension – 'figural goodness'" (Hochberg & Brooks, 1960, p. 354). This runs counter to the comparison of the induced depth effects in Fig. 11: The line drawings of cubes in a and b have a much stronger effect than those in e and f. Which implies that the interaction between the components identified by Hochberg and Brooks (1960) is not additive but that they have to be modelled as competetive processes producing the maximum joint effect if they all are of comparable magnitude. That this interpretation is not restricted to the exemplars in Fig. 11 can be illustrated by the analysis of generalizations of wire-cube drawings: (i) Repetitions and glide symmetries can be used to produce tilings with the Necker cube as constituting elements (Fig. 14), (ii) the dimensionality of the generating spatial object can be increased from 3 to 4; a 4-dimensional hyper-cube is defined as consisting of 32 edges and 16 four-fold vertices (see figures 15 & 16), or (iii) the cube as one exemplar of the Platonic body can be exchanged against a more complex one, the dodecahedron (Fig. 17).

In all cases, the factor-analytic criteria plus the convexity criterion of the generalized soap bubble would predict that the spatial effect should even be stronger than that in Fig. 11b. By inspection, it can be seen that this is

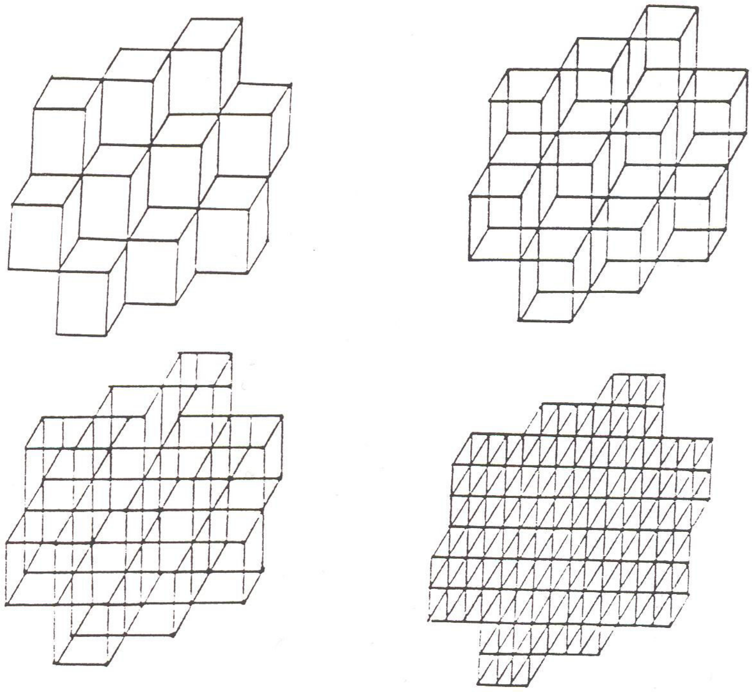


Fig. 14 Tilings consisting of Necker cubes which induce a depth effect and multistability only on a low level of complexity

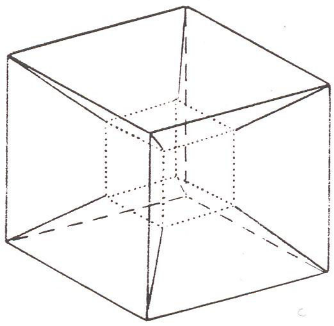


Fig. 15 A hypercube projected into 2 dimensions with 16 fourfold vertices

not the case and experimental results concerning the depth effect after an occlusion of the vertices support this.

In regard to the tilings in Fig. 14 one could argue that due to multistability different perspective orientations cancel out the over-all depth effect, however, such different perspective orientations at the same time do not occur: If one segment switches, all other become enslaved at such a speed that

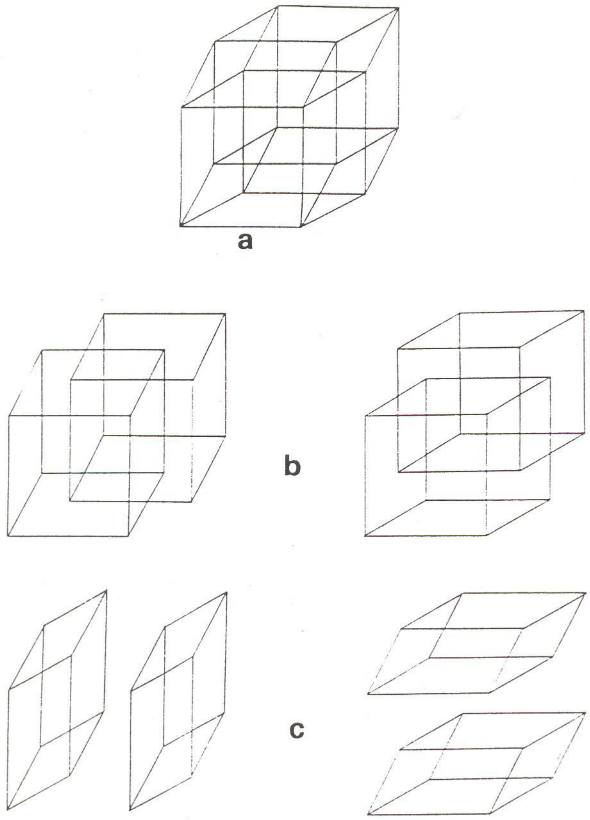


Fig. 16 A different partial view of the 4-dimensional Necker cube which does not exhibit multistability and induces only a weak depth effect. However, if disected into constituting parts (parts b and c show two alternate ways to do it), the parts exhibit multistability in depth

selfobservation cannot determine the exact and detailed time course of the process or the subprocesses of enslavement.

However, the theory of tilings offers one alternative interpretation: The patterns in Fig. 14 are all periodic, that is, there exists a subpattern which is repeated over and over resulting in glide symmetries; if the periodicity is broken, as in the non-periodic tiling of Fig. 8, multistability is given in spite of the complexity in the pattern. My conjecture is that aperiodicy, convexity, and symmetry breaking are the decisive factors for the depth effect in drawings.

In the following part I want to show how artists have made use of these factors in producing strikingly realistic pictures which at closer inspection deviate from a direct projective transformation as implied by the devices for perspective drawing which Alberti and Dürer invented. These deviations

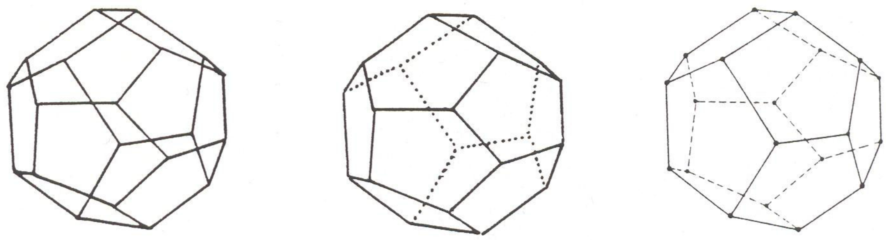


Fig. 17 Dodecahedrons which induce only a strong depth effect – but no multistability – if the lines in front and in the back are different

accentuate especially the factors of symmetry breaking and convexity whereas non-periodicity is usually already given by the chosen subject.

4 What is really perceived: Neither perspective nor an arrangement of immutable objects

In his analysis of art and geometry, Ivins (1946/1964) observes that the perception of space can be visual as well as tactile: "Tactually, things exist in a series of *heres* in space, but where there are no things, space, even though "empty," continues to exist, because the exploring hand knows that it is in space even when it is in contact with nothing. The eye, contrariwise, can only see *things*, and where there are no things there is nothing, not even empty space, for that cannot be seen. There is no sense of contact in vision, but tactile awareness exists only as conscious contact." (p. 5). If the representation what the "hand" perceives, there results absolute size and form constancy; if, however it is founded on the perception by the eye, the transformations of affine geometry, deformation and foreshortening, result. For Ivins "...intuitionally the Greeks were tactile minded, and when ever they were given the choice between a tactile or visual way of thought they instructively chose the tactile one." (p. 6). He explains this as being due to the fact that even in the "great period" the Greeks were primitive people, implicitly equating geometric competence with primitiveness.

Ivins equates the progress in visual art with the observation of perspective. However, as simple as this equating suggests, the role of perspective in art is far from straightforward. The inventions of Alberti's and Dürer's window have been heralded as at the same time the foundation of projective geometry and as the breakthrough in art for the naturalistic representation of the world. The first proposition might be true, the second is patently not as can be seen by comparing the projections of Dürer's window (Fig. 18 a,b) with his drawings (Fig. 18c).

In a way, in his drawings Dürer has disambiguated what in proper projections becomes ambiguous by adhering to form constancy, as can be seen

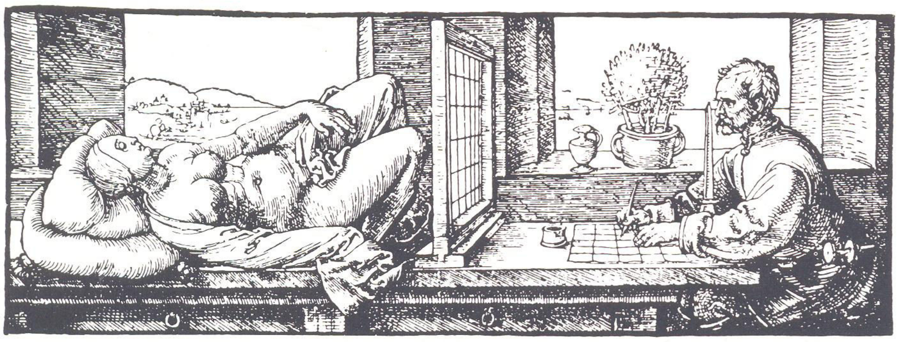


Fig. 18a The apparatus of the Dürer window (from Dürer's Unterweyssung der Messung, 1538)

Fig. 18b What a projection would look like

in Fig. 20 where different forms induce the same projection on the retina of the eye; in contrast Dürer has kept similar forms as similar as possible even if the projections are different and vice versa.

The comparison of the reclining body of Christ by Dürer (1505) and by Mantegna (after 1501) makes apparent how much closer Mantegna adheres to what is seen in a device like the Alberti window but even he 'corrects' the projection towards a greater form and size constancy (Fig. 20a,b).

How the combined manipulation of size and form constancy induces striking spatial effects has been well known in Italian Renaissance architecture, a

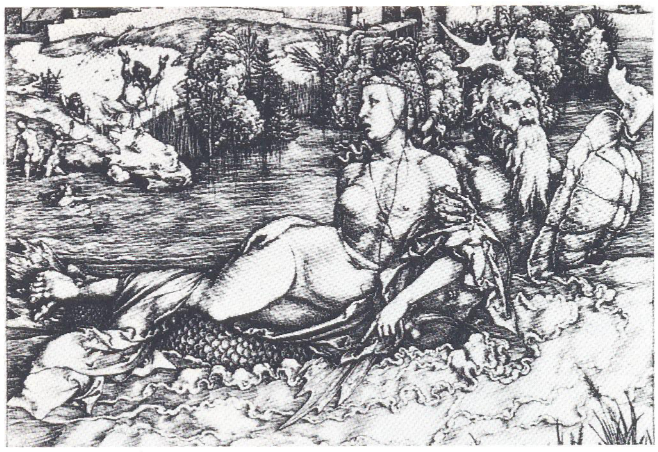


Fig. 18c What Dürer has produced under comparable perspective conditions (Das Meerwunder, 1498)

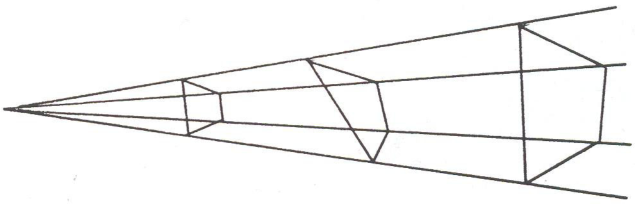


Fig. 19 Different quadrangular forms giving rise to the same projection on the retina

good example being the interior of the transept tower of S. Sidone in Torino (Fig. 21).

How this effect is diminished when different forms are used can be seen in the picture of the cupola of S. Carlo alle quattro fontane in Rome (Fig. 22) supporting the result of the preceding section according to which complexity reduces the spatial effect. This in turn sheds light on the limitations of space perception due to the processing demands.

The position implicit in Ivins' treatise of 'Art and Geometry' is that visual art in obeying the laws of projective geometry produces pictures which give the veridical account of the physical world from a specific point of view. The comparison of the projections in the Dürer window with the actual drawings and paintings of Dürer, however, makes apparent that only by means of systematic deviations from the results of projective geometry can the artist induce a veridical representation in the perceiver. How extreme these corrections sometimes have to be can be shown in Raphael's 'School of Athens' (Fig. 23a) where Euclid and another person carry perfect spheres which, as

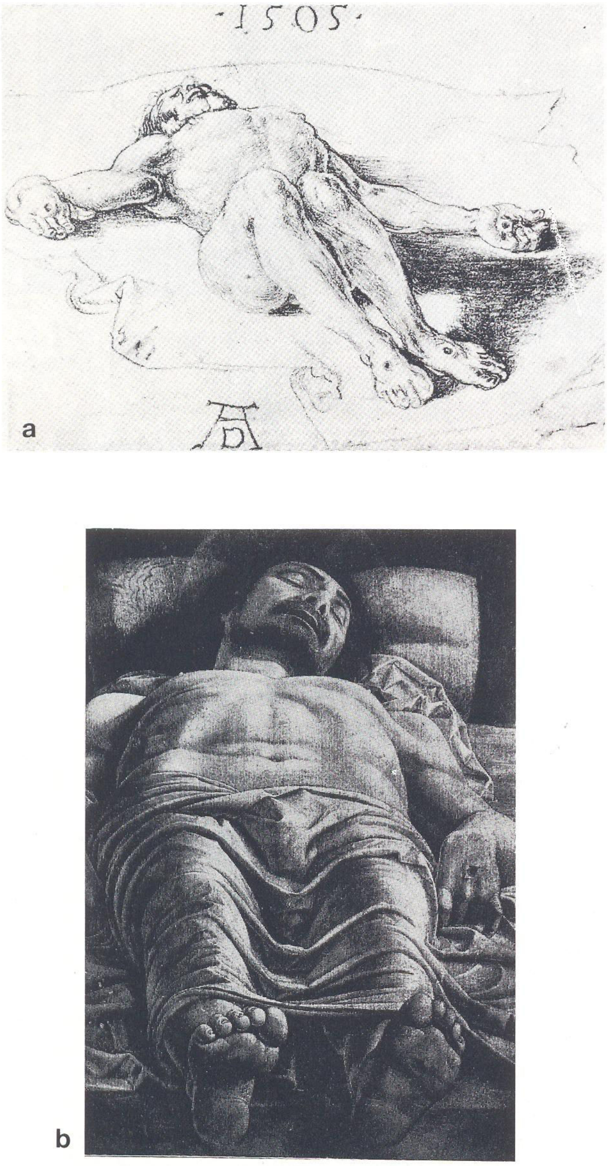


Fig. 20 Two views of the reclining body of Christ a) by Dürer (1505), b) by Mantegna (after 1501)


a


b

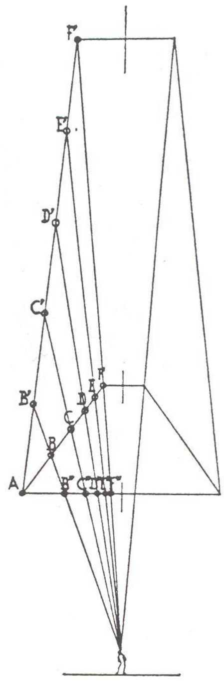

c

Fig. 21 The tower of S. Sidone, Torino a) view of the tower from inside b) view from outside c) constructive principles

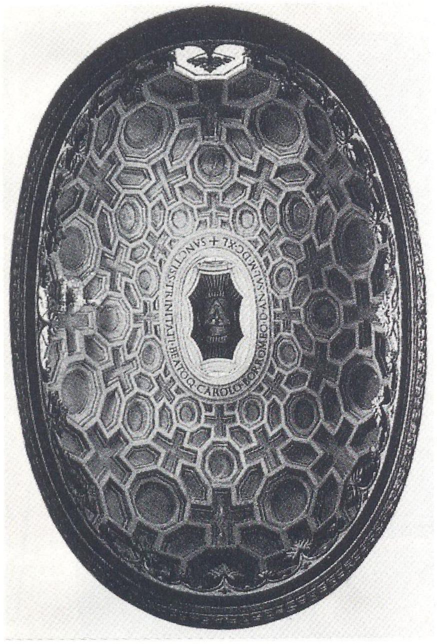
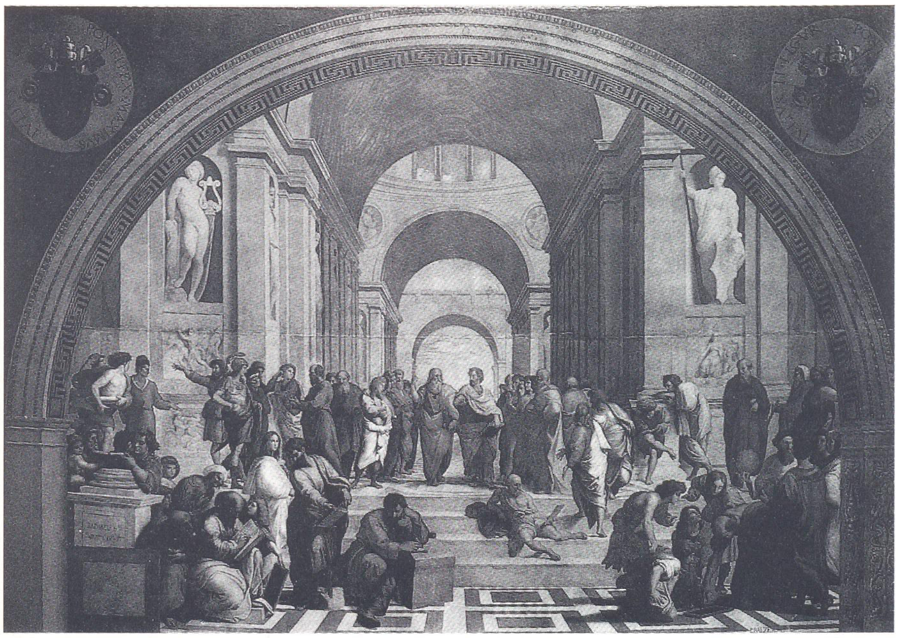


Fig. 22 The cupola of S. Carlo alle quattro fontane, Rome

LaGournerie (1859, p. 170) has demonstrated, should have the appearance given in Fig. 23b.

LaGournerie had the circles changed into the perspectively correct ellipses and presented them to colleagues who found them quite unacceptable. "They did not at all look like spherical objects – whereas Raphael's do so" (Pirenne, 1970, p. 122). The spheres and all the other figures are not drawn according to the rules for central projections. Each is drawn from one specific subsidiary center of projection just in front of the figure. Pirenne (1970, p. 122 – 123) comments on this for an Empirist puzzling result as follows: "Thus it appears that the spectator looking at Raphael's picture of the spheres must make a complicated intuitive compensation. On account of natural perspective, the circles appear foreshortened to him. They do not form in his eyes the retinal images which would be formed by actual spheres. But, on the basis of his knowledge of the shape and position of the surface of the painting, he recognizes them as circles drawn on a flat surface. Since real spheres always look circular, he concludes that these circles represent spheres. It will be noted that all this, which must somehow occur unconsciously, can be done as well when the spectator uses both eyes, and is in the wrong position. To most spectators, the *School of Athens*, in which the perspective is in parts inaccurate, appears as an outstanding example of the use of perspective." However, the 'complicated intuitive compensation' on behalf of the spectator



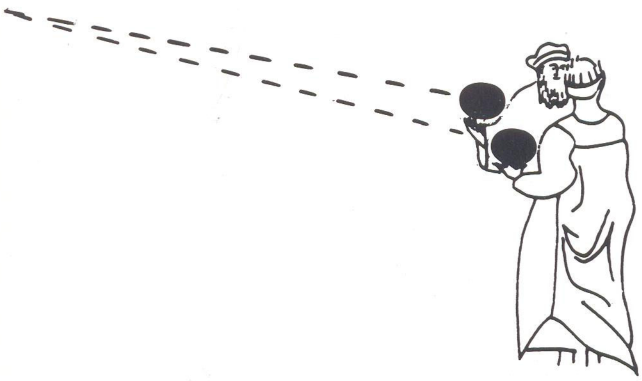


Fig. 23 (a) Raphael's School of Athens; (b) Euclid and companion with perspectively correct spheres

is effortless and as fast as rejecting the ellipses as out of place; therefore it indicates automatic, not analytic processes. The way spatial depth and constancy of form are at the same time present in the School of Athens implies two theoretical results: (i) perception is top-down and bottom up at the same time, and (iii) the constituents of complex scenes are processed in a piecemeal fashion, glance for glance with every glance having its owen frame of reference.

This conception of perception as the automatic integration of glances has been proposed by Hochberg (1962); it accounts not only well for the form constancies as in Raphael's School of Athens, furthermore it makes plausible why these regions which are especially attended to are perceived enlarged in comparison to those not, or less, attended to, and finally it explains why the patent inconsistencies in forms like the Penrose triangle are not perceived – not even by scrutiny – but have to be analyzed.

The distribution of attention while perceiving a scene or a picture is determined by bottom-up as well as by top-down processes: Before the information in the visual pathway has been analyzed in the visual cortex and in the parietal lobes it already triggers and directs the saccades in the superior colliculus by which the next glance is sampled; superimposed on these saccades are the voluntary eye movements being either the result of the analysis of the information in the visual pathway, the conscious percept, or of the intentional state the perceiver is in. As an aside it should be remarked here that Wundt (1874) had already found that there is no one-to-one relation between eye movements and the focus of attention but the correlation is high. The result of a differential sampling of glances in a percept is illustrated in Fig. 24 a–c.

Figure 24c gives only a rough approximation of the percept: Inside of each glance linearity, orthogonality, and form constancy are preserved, therefore the curvilinearity in the Escher drawing is not what is normally perceived. That the percept really is made up by glances, Hochberg (1962) has shown in his analysis of Escher prints of impossible scenes; another example is R.N. Shepard's 'Doric Dilemma' (Fig. 25) where it is obvious that the bases and the capitals are not sampled in the same glances.

Something additional about this notion of perception-frame-glances can be derived from the perception of this Fig.: The puzzlement induced by this drawing is due to the lack of an explicit boundary between the conflicting upper and lower part of the temple. The conceptual distinction between a square base and discrepant upper part with fitting capitals does not induce a perceivable edge – a result very much in accordance with Kanizsa's (1955) analysis of illusory contours which are neither due to top-down nor to bottom-up processes but the result of the interactions (or the resultant forces) in the perceptual field.

For the central perspective in Raphael's most famous paintings the piecewise corrections for form and also for size are mandatory – making use of and being necessitated by the glance-to-percept characteristic of human vision – however, this is not the cause for the impression that the depth appears flatter than intended by Raphael in contrast to the immediate and striking impression of depth one finds in the Illusionist paintings of the Baroque period, most famous Pozzo's painting on the ceiling of S. Ignazio in Rome (see Fig. 26 a & b).

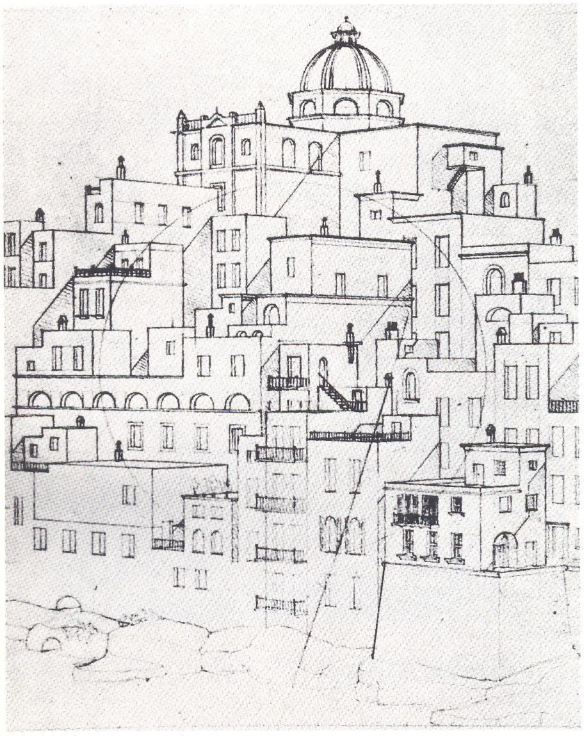


Fig. 24a The undistorted view

Whereas the Classicist ideal in the Renaissance implied the central perspective, in the Baroque period artists liberated themselves from that iconographic convention and made use of all possibilities of projective geometry (e.g. 3 vanishing points); this progress is obvious when one compares scene designs of the respective periods (Figs. 27a,b).

The design by Serlio resembles in its effect more perspective tilings (see Fig. 16) than a scene in depth, but a nearly irresistible trompe d'oeil effect is only achieved if as in Fig. 28b the symmetry is broken, a principle already to be found in the foregoing analysis of the Necker cube. A combination of these techniques, symmetry breaking, form and size constancy is characteristic for the Veduti di Roma by Piranesi.

The phenomenon of mixed perspectives, that is, using for the different parts in a picture different vanishing points, and that of parallel projection have been interpreted as indicators for a lack of mastery in techniques of perspective and projective geometry in general. However, if perception is governed by the complementary tendencies towards stability and singularity, no singular point of view in a picture can induce an impression which mimicks perception in a natural scene. What distinguishes the comparatively crude

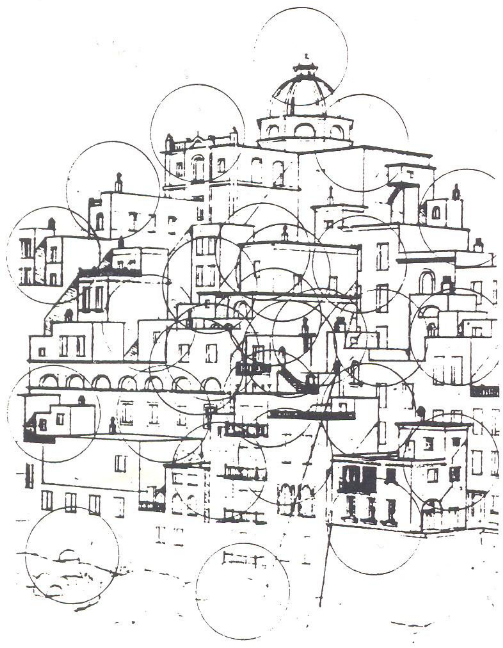


Fig. 24b A possible distribution of glances (circles)

16th century view of Amberg's castle (Fig. 28) from Piranesi's Veduti di Roma (Fig. 29) is not the veridicality of the depiction as measured by the strict adherence to the laws of projective geometry but the excelient technique of Piranesi in making the switching from one vanishing-point view to the other or from one size frame to the other smooth and imperceptible.

Figure 29 shows the Forum Romanum (Campo Vaccino) by Piranesi. Levit (1976) has taken a photograph from the same window from where Piranesi had made a sketch with a comparable angular aperture (Fig. 29).

Figure 30 shows where Piranesi deviates systematically from projective geometry: He accentuates the size of objects which can serve as points of orientation (e.g. the Collosseo), he shifts the orientation of the arch making it more stable, and he induces the impression of singularity by shifting and enlarging the ruins of the temple in the foreground. In a conversation with his friend Hubert Robert, Piranesi remarked that the sketches of the city served him only as memorizing device, the real sketch he had in his head. The comparison of the Campo de Vacco as depicted by Piranesi with Levit's photograph who took the same ventage point from the Capitol and the same angular aperture shows 3 specific systematic distortions Piranesi applies:

(i) *increasing the size* of objects which serve as orienting points for the viewer,
(ii) *shifting and rotating objects to* optimize the spatial effect, and
(iii) *changing the height/width relation.*

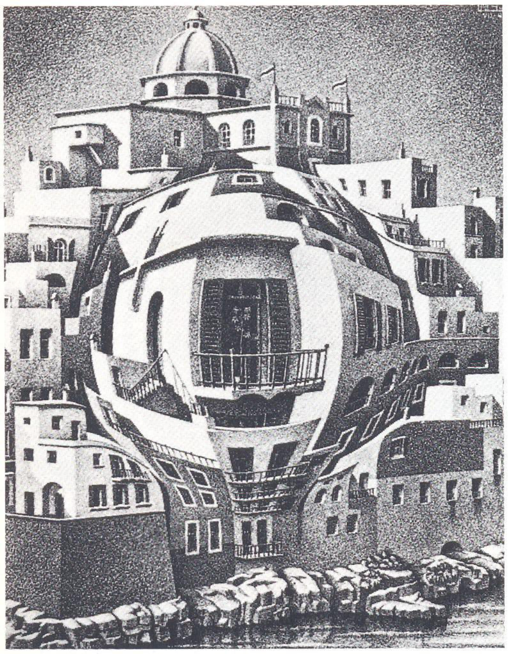


Fig. 24c The resulting percept (after Escher)

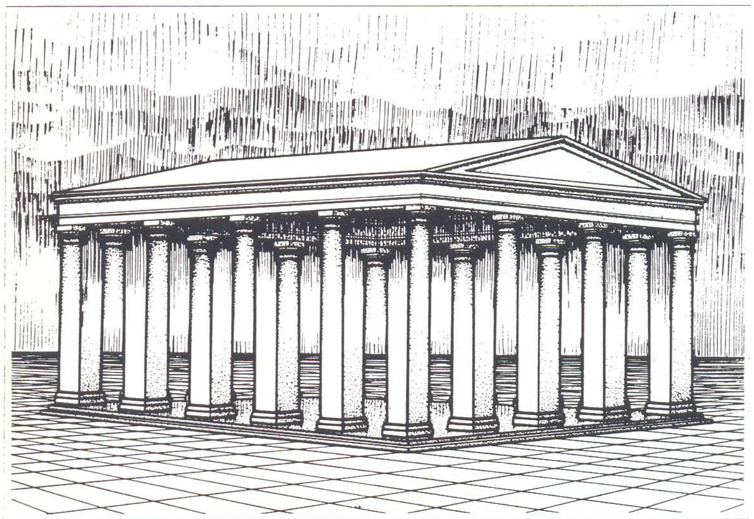


Fig. 25 R.N. Shepard: The Doric Dilemma

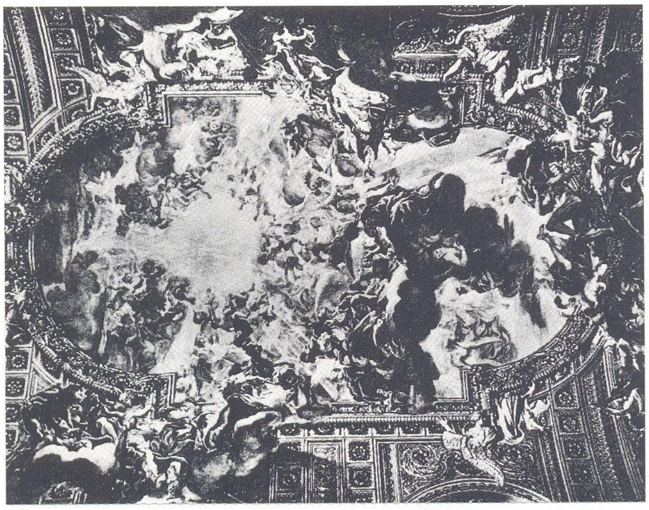


Fig. 26a Pozzo's painting

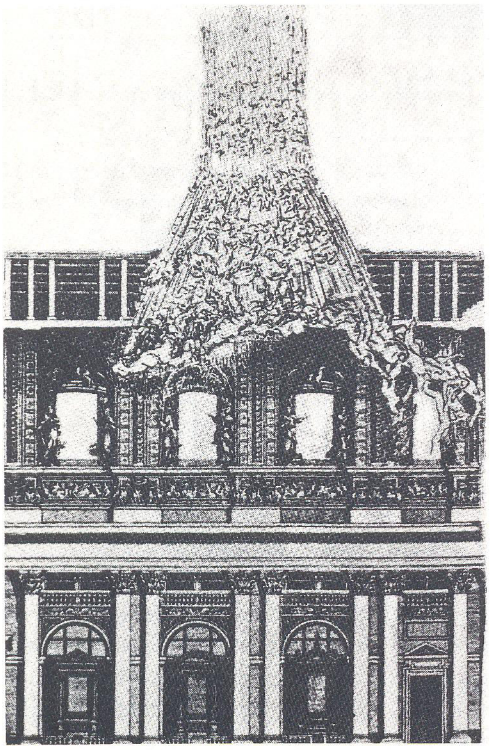


Fig. 26b The architecture implied in Pozzo's painting (after Uttal)

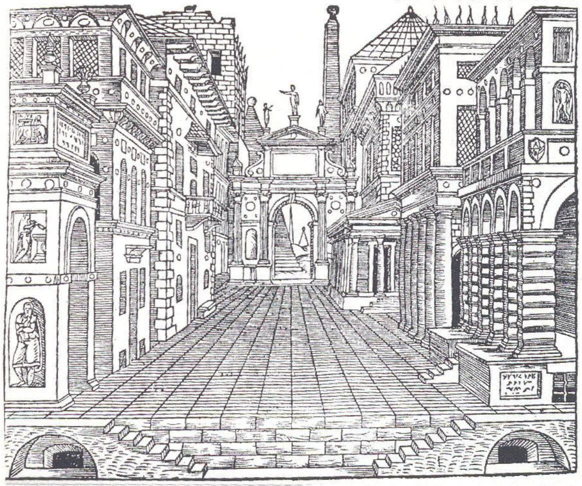


Fig. 27a Scenic design by Serlio (1551)

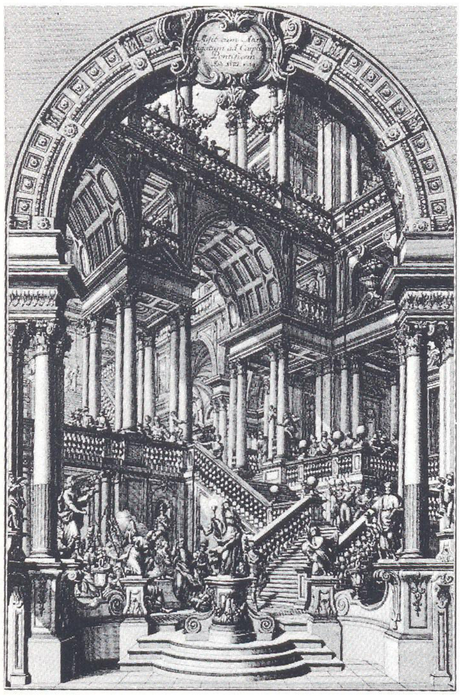


Fig. 27b Bibiena (1740)

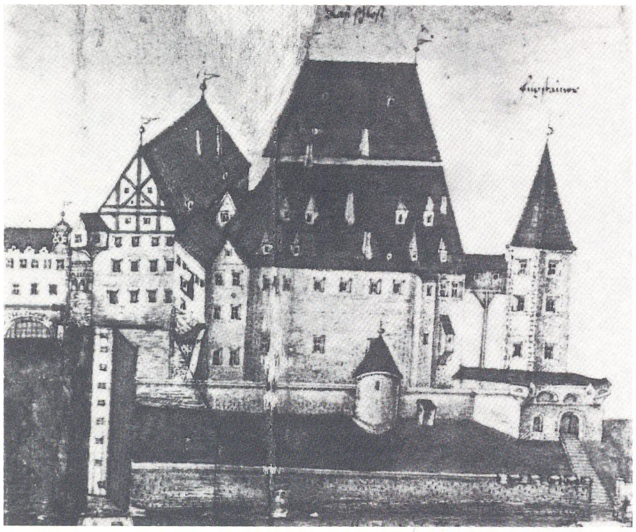


Fig. 28 Part of a Renaissance aerial view of the city of Amberg (1598) showing the castle

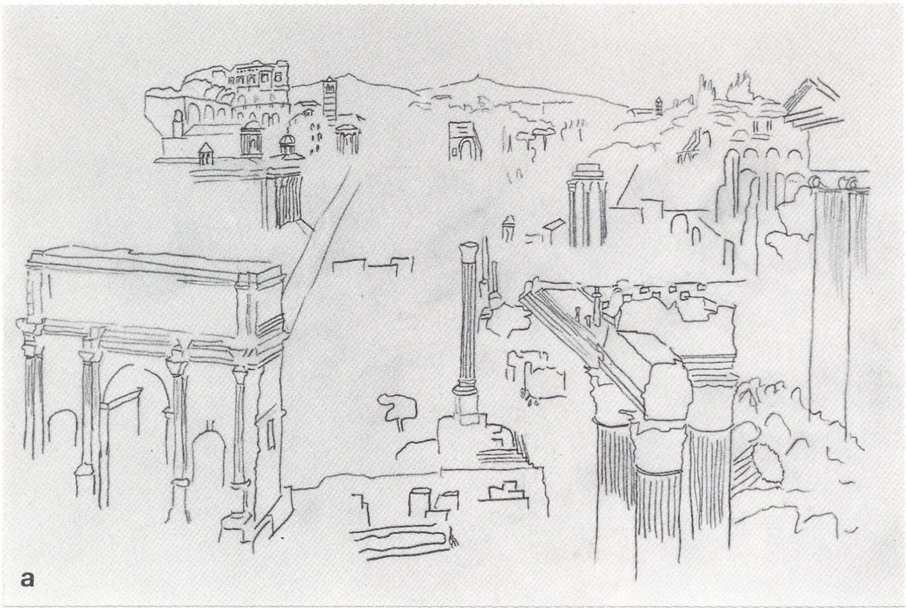
Fig. 29 G.B. Piranesi 'Veduta di Campo Vaccino' (after 1748) from 'Vedute di Roma'

Fig. 30 Levit's (1976) photograph of the Forum Romanum showing the same scene as in Fig. 29

In order to pinpoint these techniques I have made comparable line drawings from the Piranesi etching as well as from the Levit photograph. Where the excavation since the last century have changed the location I have taken from Piranesi only the still extant objects and have added to them what is now visible. Thus I have produced two drawings which differ only in the systematic distortions used by Piranesi (Fig. 31a,b).

Showing these drawings to tourists which had visited Rome (n=32) with the question which drawing depicted most correctly the real scenery, resulted in a majority (23) for the Piranesi-type drawing, 7 being undecided and only 2 voted for the perspectively correct drawing. This makes clear that Piranesi's distortions are not ideosyncratic whims but that 'in his head' the same accentuating processes took place as in the normal perceiver. This combination of exact observation and 'reconstruction out of his head' allows Piranesi even to show his scenes from that point of view which induces best the intended effect even if it is physically not accessible.

The comparison of the views depicting 3 Napolitan baroque churches of comparable structural complexity makes apparent how important the chosen point of view is for the induction of a spatial impression: (i) The fronto-parallel view of S. Anna a Porta Capuana (Fig. 32a) leaves the symmetries in



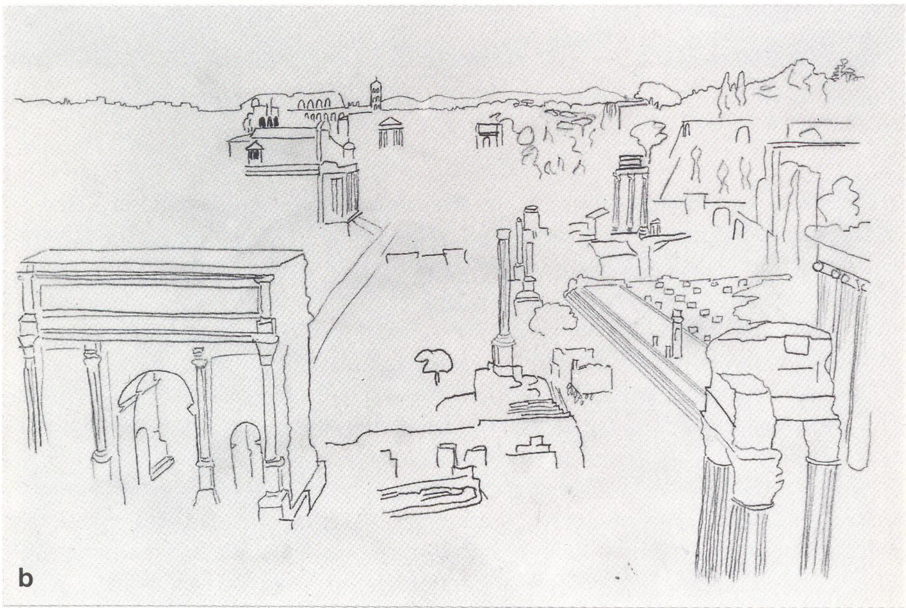


Fig. 31 (a) Line drawing after Fig. 29; (b) line drawing after Fig. 30

the architectural design unchanged, the induced spatial effect is negligable, if existent at all. (ii) The view from below of S. Maria della Sanità (Fig. 32b) tries to mimic the view as given when entering the church from the side porch; that there is an induced depth effect can be seen from the fact that in the cupola and in the apse a concave-convex bi-stability occurs; this, however, impedes the formation of a global 3-D effect. (iii) The view from above of Concezione a Monte Calvario (Fig. 32c) induces a strong and globally consistent 3-D effect.

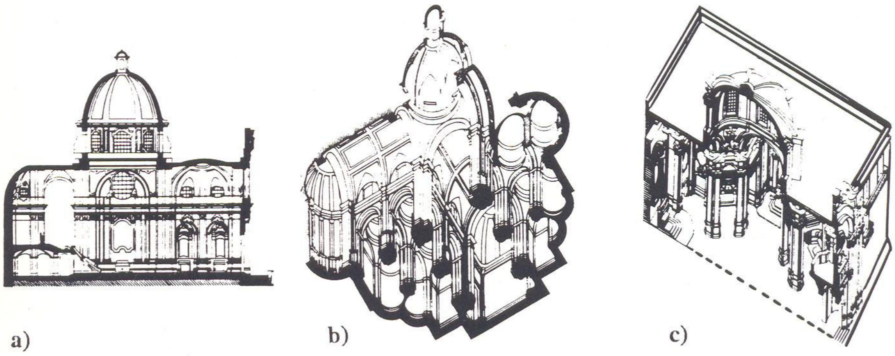


Fig. 32 Line drawings of three Napolitan Baroque churches; (a) S. Anna a Porta Capuana, (b) S. Maria della Sanita, (c) Concezione a Monte Calvario

All three views have in common that they are parallel projections, a fact which obviously does not impede a 3-D effect if the dominant symmetries are broken; this together with the strong global spatial impression of Fig. 32c which is taken from an inaccessable and for normal surface-bound perception unusual vantage point implies that the 3-D effect in pictures is not due to conventions of perceiving acquired through experience with the rules of depiction, but to strong local processes of perception which interact, sometimes in a competetive way as in Fig. 32. Apparently, Piranesi in "the perfect marriage of hard fact with an incomparable poetic eloquence" (Levit, 1976 p. X.) knew about the sometimes imagery vantage point producing the strongest 3-D effect because he usually depicts the major objects from the side (15 – 30°) and slightly from above (7 – 15°).

The analysis of multistability so far supports the theoretical position that in space perception bottom-up and top-down processes interact. One could, however, argue that in the choice of examples already lies a kind of question begging, namely that in relying on specific forms experience as a top-down process necessarily plays a role in constructing depth in 2-D displays. That form perception in itself is at least partially inborn has been shown by Fantz (1961), but even for the prototypical example of a bottom-up processing of information, namely the perception of colour, it can be shown that only in the case of experimenting with colour chips in isolation pure bottom-up processing can be assumed; if the complexity of the perceptual task is increased, top-down influences can be observed. It is interesting to note that in the history of art (Ruskin) and in the theory of art (Mondrian) radical positions of bottom- up processing in colour vision have been proposed, developed independently from psychological theories of perception and very much earlier than Marr's influential treatise 'Vision' (1982).

J. Ruskin (1857) in writing about William Turner presents his bottom-up theory as follows: "The perception of solid form is entirely a matter of experience. We see nothing but flat colours; and it is only by a series of experiments that we find out that a stain of black or grey indicates the dark side of a solid substance, or that a faint hue indicates that the object in which it appears is far away. The whole technical power of painting depends on our recovery of what may be called the *innocence of the eye* that is to say, of a sort of childish perception of these flat stains of colour, merely as such, without consciousness of what they signify, – as a blind man would see them if suddenly gifted with sight." (XV, p. 27).

Ruskin's speculations are – at least partially – supported by clinical data: Gregory and Wallace (1963) report the case of S.B, born blind in 1906 and given a corned graft in 1959 which allowed him to see: "The difficulty with pictorial representations was general, and it persisted; he did not respond to the common geometric illusions, and perspective seems not to have suggested depth to him. He never learned to read properly" (Morgan, 1977, p. 182).

In 1958 Mondrian has described his programmatic stance from a comparable Empiricist position:

"In cubism abstraction has not been developed until its final goal: The expression of pure reality I felt that this reality could only be achieved by pure construction. In its utmost expression pure construction is independent from subjective emotions and imaginations. It took a long time until I realized that the specific form and the natural colour induce subjective mental states which obscure pure reality. If one wants to construct the Gestalt of pure reality, it is necessary to reduce natural forms to the constant elements of form and the natural colours to the elementary colours" (1958, p. 9 translation by the author)

However, that this is not the only possible interpretation becomes clear when we turn to Turner on whose paintings Ruskin founded his cited position. Turner regarded his painting as an explicit application of Goethe's theory of colour which he commented in writing and painting, e. g. §808 of 'Zur Farbenlehre' (1810) "Is the totality of colour like an object brought to the eye from the outside, then it is agreable to it because the sum of its own activity approaches it as reality" is commented by Turner in the following way - "this is the object of Paintg (painting)". His own paintings defy Ruskin's concept of the "innocence of the eye" which perceives colours and not meaning, especially his illustrative sketches for Goethe's theory of colour.

Turner was the first to use colour not to enhance the geometrical perspective induced by foreshortening but as a means in itself to give a 2D display the impression of a third dimension; prototypical examples are his 1819 water colour picture "Colour Beginning" and his "Light and Colour (Goethe's Theory)" of 1843. Goethe himself in §155 of his "Zur Farbenlehre" (1810) has described the spatial impressions related to the colour blue: "...In the mountains during the day the sky appears to show a Prussian blue because only a very faint mist have before the infinite dark space; when descending, the blue becomes lighter until it changes entirely into a whitish blue in certain regions when the density of the mist increases." Especially Turner's "Colour Beginning" shows that the spatial impression due to colour is not confined to different densities of mist. As for instance Wolfgang Metzger (1975) has observed any colour induces a typical distance with blue being the farthest away. But Turner systematizes the spatical impression of colour further: He is the first to analyze systematically the conditions of transparency as shown in his water colours where he depicts overlapping colours. Insofar there is a direct tradition of research starting with Goethe's theory of colours, taken up again by Metelli (1975), and analyzed in formal approaches by Beck et al. (1984) and Gerbino (1988). This tradition highlights the fact that contrary to the positions taken by Ruskin or Mondrian and by Marr's bottom-up theory (1982), perception it is not a sequential process which starts with colours, subsequently derives forms, and finally constructs space but a parallel process tuned for 3-dimensionality.

5 Conclusion: By exploring the role of multistability we realize that reality and actuality meet

The demonstrated phenomena of multistability illuminate one important aspect about the relation between reality (the transphenomenal world of physical objects) and actuality (the mentally represented world which directs actions and interprets the results of them): It is not only many-to-one (the classification of objects and events into equivalence classes, Aristotles' problem) but also one-to-many (identical stimuli can give rise to different percepts). This indeterminacy in the relation between the physical world and the mental representation with its intentionality does not imply an anythinggoes stance because in the cases of multistability usually only small number of attractors exist what, however, is principally unpredictable is the exact basin in which stability will be reached. That is, the physical world constrains but does not determine the actuality of the mind.

The notion of competing processes in perception which reach points of stability, that is, an equilibrium where the effective forces are balanced, appears to bridge the gap between the concept of a perceptual field, central for Gestalt psychologists as Köhler (1920) and Koffka (1935), and that of parallel distributed processes as developed by Rumelhart and McClelland (1986). At the same time it helps to clarify the notion of a complementarity between stability and singularity in perceptual and cognitive processes: The combination of effective forces characterizing a stable percept is achieved by fixing each of the competing processes at a point which is singular and therefore not a stable basin. The standard example for this is the Necker cube which exhibits its spatial impression only if neither the overall symmetry nor the similarity with the geometric characteristics of the cube are maximal. Insofar, in the perception of even moderately complex scenes (e. g. the Necker cube, the Schröder stairs, or the Thiery Fig.) I regard the role of the so-called Gestalt laws of perception as processes pitched against each other not as adding up in their effects. This is even to a higher degree characteristic for the more complex scenes we come across in everyday-day life; that in these cases the multistability inherent in competing processes is not experienced can be attributed to the high complexity regarding the number of processes involved and of local multistability patterns possible.

My conjecture about the relation between structural or geometric multistability and spatial perception is that space perception under normal conditions, that is, during and by perambulation, means construction and disambiguation of spatial relations at the rate prescribed by the speed of perambulation. The spatial invariants enabling the perceiver to build up a stable spatial frame of reference therefore consists of the affine transformations and not of the objects and their relative distances. This process is mimicked by multistability. However, the closer analysis of the conditions for multistability reveals that the extraction of affine transformation is in turn constrained by temporal and/or spatial frequencies and the complementarity of symmetries and complexities of scenes and objects. As demonstrations can serve the tilings in Fig. 15 and the 2-D projection of a 4-D Necker cubes in Fig. 16.

Shepard (1987) has summarized the epistemological stance I have called Interactive Realism very succintly in the following way: "I have argued that to the extent that the principles of the mind are not merely arbitrary, their most likely ultimate sources are the abiding regularities in the world.... Among such external regularities, the most abiding are the ones that in the long run should have the greatest opportunity to become internalized – however abstract those regularities may be:....[t]he facts that space is three-dimensional, that objects have six degrees of freedom of global motion, that light and darkness alternate with a fixed period, and that sets of objects having the same significant consequences tend to form a compact region in an appropriate parameter space ... (p. 269)."

References

Attveave, F. (1981): Prägnanz and soap bubble systems: A theoretical exploration. In: J. Beck (Ed.), Organization and Representation in Perception. Hillsdale, NJ: Lawrence Erlbaum Ass.

Beck, J., Prazdny, K. & Ivry, R. (1984): The perception of transparency with achromatic colours. Perception and Psychophysics 35, 407 – 422.

Borst, A. & Egelhaaf, M. (1992): Im Cockpit der Fliege. MPG Spiegel 3, 14 – 16.

Bradley, D.R. & Petry, H.M. (1977): Organizational determinants of subjective contour: The subjective Necker cube. American Journal of Psychology 90, 253 – 262.

Brandimonte, M.A. & Gerbino, W. (1993): Mental image reversal and verbal recoding: When ducks become rabbits. Memory and Cognition 21, 23 – 33.

Fantz, R.L. (1961): The origin of form perception. Scientific American 204, 66 – 72.

Fodor, J.A. (1974): The Language of Thought. New York: Crowell.

Gerbino, W. (1988): Models of achromatic transparency. Gestalt Theory 10, 5 – 20.

Gibson, J.J. (1967): New reasons for realism. Synthese 17, 162 – 172.

Goethe von, J.W. (1810): Zur Farbenlehre. Tübbingen: Cotta.

Goldmeier, E. (1982): The Memory Trace: Its Transformation and its Fate. Hillsdale, NJ: Erlbaum.

Gregory, R.L. & Wallace, J.G. (1963): Recovery from Early Blindness. EPS Monograph, No. 2. Cambridge: Heffer.

Haken, H. (1991): Synergetic Computers and Cognition. Berlin: Springer.

Hochberg, J. (1962): The psychophysics of pictorial perception. Audio-visual Communications Review 10 (5), 22 – 54.

Hochberg, J. & Brooks, V. (1960): The psychophysics of form: Reversible-perspective drawings of spatial objects. American Journal of Psychology 73, 337 – 354.

Ivins, W.M. jr. (1946): Art and Geometry. A Study in Space Intuitions. New York: Dover Publications.

Kanizsa, G. (1955): Condizioni ed effetti della transparenza fenomenica. Rivista di Psicologia 49, 3 – 19.

Kanizsa, G. & Luccio, R. (1986): Die Doppeldeutigkeiten der Prägnanz (transl. A. Hüppe & M. Stadler). Gestalt Theory 8, 99 – 135.

Köhler, W. (1920): Die physischen Gestalten in Ruhe und im stationären Zustand. Erlangen: Verlag der Philosophischen Akademie.

Koenderink, J.J. (1990): The brain a geometric engine. Report Nr. 19/1990 Research Group on Mind & Brain, ZiF Bielefeld.

Koffka, K. (1935): Principles of Gestalt Psychology. New York: Harcourt Brace.

Land, M.F. & Fernald, R.D. (1992): The evolution of eyes. Annual Review of Neuroscience 15, 1 – 29.

LaGournerie, J. de (1859): Traité de perspective linéaire contenant les tracés pour les tableaux plans et courbes, les bas-reliefs et les décorations théatrales, avec une théorie des effets de perspective, vol.1 and atlas of plates. Paris: Dalmont & Dunod; Mallet-Bachelier.

Leeman, F., Elffers, J. & Schuyt, M. (1976): Hidden Images. New York: Harry N. Abrams.

Levit, H. (1976): Views of Rome Then and Now. New York: Dover Publications.

Li, W., Packard, N.H. & Langton, Ch.G. (1990): Transition phenomena in cellular automata rule space. In: H. Gutowitz (Ed.), Cellular Automata, pp. 77 – 94. Amsterdam: North-Holland.

Marr, D. (1982): Vision. New York: Freeman

Maturana, H.R. (1982): Erkennen: Die Organisation und Verkörperung von Wirklichkeit. Braunschweig: Vieweg.

Metelli, F. (1975): The perception of transparency. In: G.B. Flores d'Arcais (Ed.), Studies in Perception: Festschrift for Fabio Metelli, pp. 445 – 487. Milano: Martello-Giunti.
Metzger, W. (1975): Gesetze des Sehens. Frankfurt: Kramer (3rd ed.).
Michaels, C.F. & Carello, C. (1981): Direct Perception. Englewood Cliffs, NJ: Prentice Hall.
Mondrian, P. (1958): Lebenserinnerungen und Gedanken über die "Neue Gestaltung". Das Kunstwerk 11, 9 – 10.
Morgan, M.J. (1977): Molyneux's Question. Cambridge: Cambridge University Press.
Perkins, D.N. (1968): Cubic corners. Quarterly Progress Report 89, M.I.T. Research Laboratory of Electronics, pp. 207 – 214 (Reprinted in Harvard Project Zero Technical Report No. 5, 1971).
Pirenne, M.H. (1970): Optics, Painting and Photography. Cambridge, MA: University Press.
Pylyshyn, Z. (1984): Computation and Cognition. Cambridge, MA: MIT Press.
Rechenberg, I. (1990): Evolutionsstrategie – Optimierung nach Prinzipien der biologischen Evolution. In: J. Albertz (Ed.), Evolution und Evolutionsstrategien in Biologie, Technik und Gesellschaft. Schriftenreihe der Freien Akademie, Bd. 9. Hofheim: Hofheimer Druck- und Verlagsanstalt (2nd ed.).
Roth, G. (1992): Das konstruktive Gehirn: Neurobiologische Grundlagen von Wahrnehmung und Erkenntnis. In: S.J. Schmidt (Ed.): Kognition und Gesellschaft, pp. 277 – 336. Frankfurt: Suhrkamp.
Rumelhart, D.E. & McClelland, J.L. (1986): Parallel Distributed Processing: Explorations in the Microstructure of Cognition, vol. 1: Foundations. Cambridge, MA: MIT Press.
Ruskin, J. (1857/1987): The Art Criticism. New York: Da Capo Press.
Santayana, G. (1920): Three proofs of realism. In: D. Drake, A.O. Lovejoy, J.B. Pratt, A.K. Rogers, G. Santayana, R.W. Sellars & S.A. Strong (Eds.) (1968), Essays in Critical Realism. New York: Gordian Press.
Shepard, R.N. (1987): Evolution of a mesh between principles of the mind and regularities of the world. In: J. Dupré (Ed.), The Latest on the Best: Essays on Evolution and Optimality, pp. 251 – 275. Cambridge, MA: MIT Press.
Shepard, R.N. (1990): A possible evolutionary basis for trichromacy. Paper presented at the SPSE/SPIE Symposium on Electronic Imaging, Santa Clara, CA.
Wertheimer, M. (1912): Über das Denken der Naturvölker. Zeitschrift für Psychologie 60, 321 – 378.
Wulf, F. (1922): Über die Veränderung von Vorstellungen (Gedächtnis und Gestalt). Psychologische Forschung 1, 333 – 373..
Wundt, W. (1874): Grundzüge der Physiologischen Psychologie. Leipzig: Engelmann.
Zimmer, A.C. (1982): Criteria of optimality for retrieval schemes: Informativeness vs. Robustness. In: G. Luer (Ed.), Bericht über den 33. Kongreß der Deutschen Gesellschaft für Psychologie. Göttingen: Hogrefe.
Zimmer, A.C. (1982): Are some triangles heavier than others? A graviational model of form perception. Psychologische Beiträge 24, 167 – 180.
Zimmer, A.C. (1986): The economy principle, perceptual mechanisms, and automatic cognitive processes. Gestalt Theory 8, 174 – 185.
Zimmer, A.C. (1989): Gestaltpsychologische Texte - Lektüre für eine aktuelle Psychologie? Gestalt Theory 11, 95 – 121.
Zimmer, A.C. (1991): The complementarity of singularity and stability. A comment on Kanizsa & Luccio's "Analysis of the Concept of 'Prägnanz'". Gestalt Theory 13, 276 – 282.